Exercise 17

In Exercises 13 to 19, use set theoretic or vector notation or both to describe the points that lie in the given configurations.

The line passing through
$$(-1, -1, -1)$$
 and $(1, -1, 2)$

Solution

The equation for a line is

$$\mathbf{y}(t) = \mathbf{m}t + \mathbf{b},$$

where \mathbf{m} is the direction vector, b is the position vector for a point on the line, and t is a parameter. Subtract the two given position vectors to get \mathbf{m} .

$$\mathbf{m} = (1, -1, 2) - (-1, -1, -1)$$

= $(2, 0, 3)$

So the line in question can be written as

$$\mathbf{y}(t) = (2,0,3)t + (-1,-1,-1)$$
$$= (2t,0,3t) + (-1,-1,-1)$$
$$= (2t-1,-1,3t-1)$$

Since there's only one arbitrary constant t, the line is one-dimensional. The set of points on the line is described by

$$\{(2t-1,-1,3t-1), t \in \mathbb{R}\}.$$